Discussion of

Tail Risk, Robust Portfolio Choice, and Asset Prices

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  - A fat-tailed distribution...
  - **This paper:** A set of possible fat-tailed distributions! (ambiguity aversion)

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  - (3) Ambiguity $\uparrow\uparrow \implies$ Nonparticipation
  - (4) Fat tails $\uparrow \implies \partial$ Disaster exposure $/\partial$ Ambiguity $\uparrow$ (Effect 1 is stronger)

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$$\max_{\pi} \mathrm{E}[u(W)]$$
$$W = 1 + \pi x$$
$$x = \mu + \sigma \epsilon$$
$$\epsilon \sim \mathcal{N}(0, 1)$$

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    - ⋆ Evaluate outcomes under worst possible distribution

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    - Evaluate outcomes under worst possible distribution
    - Trade-off: Pessimism vs. degree of faith in the original "reference" model $\hat{P} = (\hat{p}, \hat{F}) \in \mathcal{P}$

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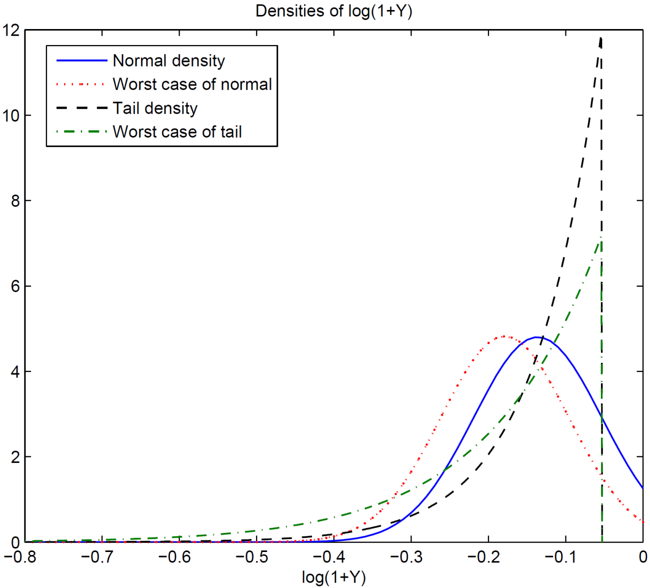
- Full model: multiple assets + continuous time with terminal date T

Key Finding: Need Both Fat Tails and Ambiguity Aversion



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From Hansen and Sargent (2008)

$$\max_{k} \min_{P\in\mathcal{P}} \left\{ E_P[u(c)] + \theta \text{ Entropy}(P, \hat{P}) \right\}$$
$$\Leftrightarrow \max_{k} \left\{ -\theta \log \mathrm{E}_{\hat{P}} \left[e^{-\frac{1}{\theta} u(c)} \right] \right\}$$

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- Do agents not know the true jump model or are they just particularly averse to jumps?
- Not a bad thing, just a helpful way to think about this if you're not familiar with robust control

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  - In calibration, how averse to ambiguity does an investor need to be to not participate? What is her worst-case model? Does it seem plausible for the marginal stock market entrant?
  - How does ambiguity about jump tail risks affect optimal risk-sharing, connecting this work to Ibragimov et al (2011)?

Conclusion

- Interesting paper; I learned a lot.
- Full set of model features seems necessary to get quantitative results
- Since crisis motivates the paper, I encourage authors to apply model to other asset classes, particularly those held by financial intermediaries.